

THE OSCILLATING CLUSTER CORRELATION FUNCTION

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1 Introduction

According to the traditional view galaxies are clustered on small scales, and on large scales the distribution of galaxies and clusters of galaxies is random (i.e. scale-free). Such a distribution is expected for the currently popular structure formation scenario based on the evolving dynamics of the dark matter during the history of the Universe. Quantitatively this behaviour is described by the correlation function of galaxies and clusters of galaxies which has a high peak at small separations but approaches zero at about $30 h^{-1}$ Mpc for galaxies and at about $70 h^{-1}$ Mpc for clusters of galaxies (in this paper we express the Hubble constant as $H_0 = h \ 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Initially these scales were thought to be the scales of the transition to homogeneity. However, in the distribution of galaxies voids of diameter up to $50 h^{-1}$ Mpc were discovered, voids in the distribution of clusters of galaxies are even larger with diameters of $100 h^{-1}$ Mpc and more. Thus it was thought that the homogeneity begins on supercluster scale.

Recent studies have shown that even on supercluster scale the distribution of galaxies and clusters of galaxies may have some regularity. First clear indication for the presence of periodicity in the distribution of high-density regions in the distribution of galaxies came from the deep pencil-beam survey of redshifts of galaxies by Broadhurst *et al* (1990). High-density regions in this survey form a regular pattern with a period $128 h^{-1}$ Mpc. Nearest peaks in the distribution of high-density regions in this survey coincide in position and redshift with superclusters of galaxies (Bahcall 1991), thus one may think that the distribution of superclusters may have some regularity. However, as no clear periodicity was found in other directions, this result was explained as a statistical anomaly (Kaiser and Peacock 1991).

Independent studies have shown that approximately on the same scale the cluster correlation function has a weak secondary maximum (Kopylov *et al*

(1988), Mo *et al* (1992), Einasto and Gramann (1993), Fetisova *et al* (1993)). This scale has been detected in the three-dimensional supercluster-void network by Einasto *et al* (1994), Einasto (1995a, 1995b). The analysis of the Las Campanas Redshift Survey has shown that the distribution of sheet-like and filamentary structures has a preferred scale about $100 h^{-1}$ Mpc (Doroshkevich *et al* 1996), also the two-dimensional power spectrum of galaxies of this survey has a peak on the same scale (Landy *et al* 1996). These results were based on small number of objects or (with the exception of the study by Einasto *et al* 1994) on two-dimensional data, thus further studies are needed.

To investigate the distribution of matter we have used a new redshift compilation of rich clusters of galaxies by Andernach, Tago and Stengler-Larrea (see 1995). This is the largest and deepest three-dimensional survey available presently. Using this dataset we have compiled a new catalogue of superclusters of galaxies (Einasto *et al* 1997a), calculated the nearest neighbour and void diameter distribution (Einasto *et al* 1997a), and determined the cluster correlation function and power spectrum (Einasto *et al* 1997b, 1997c, 1997d). Here I give a short summary of principal results of these studies.

2 The distribution of superclusters

The compilation of Abell-ACO clusters of galaxies by Andernach, Tago and Stengler-Larrea (1995) contains measured redshifts for 869 of the 1304 clusters with an estimated redshift up to $z = 0.12$. For the present analysis we used all rich clusters (richness class $R \geq 0$) in this compilation with at least two galaxy redshifts measured. Cluster distances were determined from redshifts or from the brightness of the clusters 10-th brightest galaxy, using the photometric estimate of Peacock & West (1993). The new catalogue contains 220 superclusters with at least two member clusters (supercluster richness); 25 superclusters are very rich with at least 8 members, approximately 25% of all clusters are members of these very rich superclusters.

The distribution of clusters in rich superclusters in supergalactic coordinates is shown in Figure 1. We see that the population of clusters in rich superclusters forms a fairly regular network. The rectangular distribution of clusters was noted by Tully *et al* (1992), who used the term “chessboard universe” to describe the structure. Void diameter and nearest neighbour tests indicate that the mean distance of rich superclusters across voids is about $120 \pm 20 h^{-1}$ Mpc. Poor superclusters and isolated clusters lie in the vicinity of rich superclusters, they also populate void walls but are absent in central regions of voids defined by rich superclusters (Einasto *et al* 1994, 1997a).

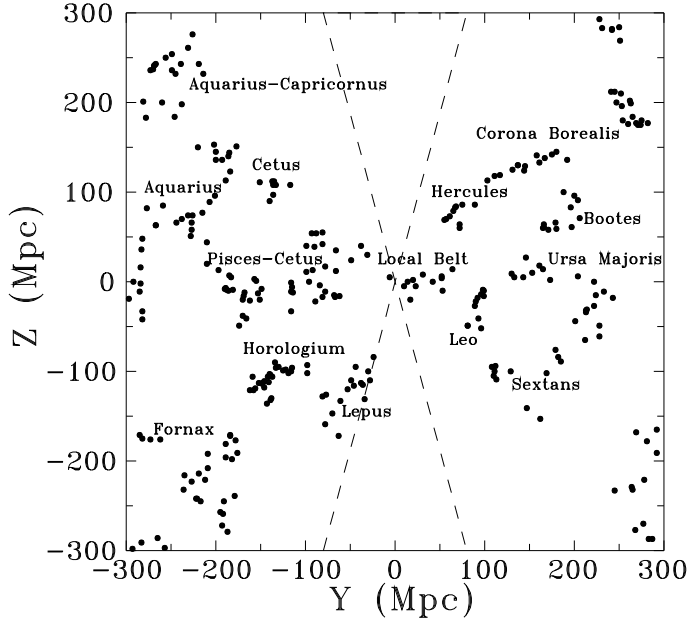


Figure 1: The distribution of clusters of galaxies in supergalactic coordinates, the sheet is taken in between supergalactic $-100 \leq X < 100 h^{-1}$ Mpc. Clusters belonging to rich superclusters (containing at least 4 clusters) are plotted with filled circles. Superclusters are identified by common names (from constellations where they are located). Dashed lines mark the zone of avoidance near the galactic plane.

3 Cluster correlation function

To quantify the regularity of the cluster distribution we have calculated the correlation function and the power spectrum for clusters of galaxies. The correlation function describes the distribution of clusters in the real space, the power spectrum in the Fourier space of density waves. Analysis of various geometric models has shown that if superclusters form a quasiregular lattice with an almost constant step size then the cluster correlation function is oscillating, it has alternate secondary maxima and minima, separated by half the period of oscillations. The period of spatial oscillations of the correlation function is equal to the step size of the distribution (Einasto *et al* 1997c). The amplitude of the power spectrum at the wavelength corresponding to that period is enhanced with respect to other wavelengths, i.e. it is peaked. In contrast, if superclusters are located randomly in space then the correlation function approaches zero level at large separations and the power spectrum turns smoothly from the region with positive spectral index on large wavelengths to a negative index on small wavelengths.

Correlation functions were calculated using the classical definition by (Peebles 1980). The cosmic variance (error) of the correlation function was determined following the method suggested by Mo, Jing and Börner (1992). The cosmic variance depends on the number of clusters in samples and does not de-

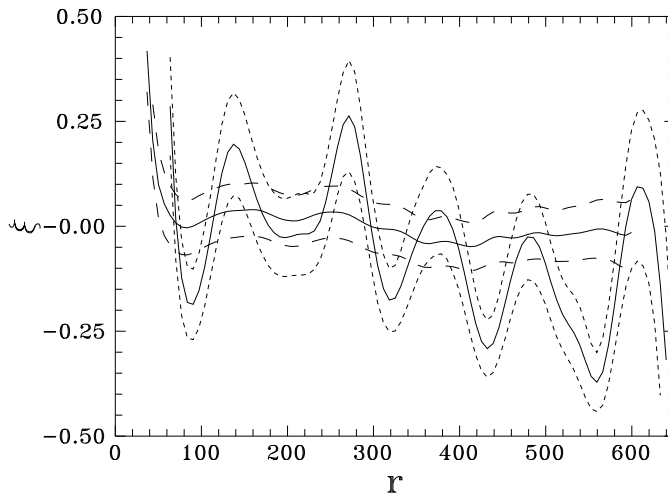


Figure 2: The correlation function of clusters of galaxies. The solid line with dotted error corridor shows the correlation function of clusters located in rich superclusters with at least 8 members, the solid line with dashed error corridor gives the correlation function of isolated clusters and clusters in poor superclusters.

pend on the bin size, the normalising constant was determined from the scatter of realisations of various N-body and geometric models (for details see Einasto *et al* 1997c).

We have divided the whole sample of clusters into two populations, one population in high-density regions (rich superclusters with at least 8 members), and the other in low-density regions (isolated clusters and clusters in poor superclusters with a number of member clusters less than 8), Figure 2. To suppress random errors the correlation function has been smoothed with a Gaussian kernel of dispersion $15 h^{-1}$ Mpc. Our results show that the correlation function of clusters in high-density regions has an oscillatory behaviour. Maxima and minima alternate with a period of $\approx 120 h^{-1}$ Mpc. The 1σ error corridor is considerably smaller than the amplitude of oscillations. A small overall decrease of the correlation function with distance is due to inaccuracy of the selection function. The population of clusters in low-density regions has an uniform correlation function which approaches zero on large scales.

The oscillatory behaviour of the correlation function is rather surprising. The presence of the first secondary maximum of the cluster correlation function was shown already by Kopylov *et al* (1988), and confirmed by Mo *et al* (1992), Einasto and Gramann (1993) and Fetisova *et al* (1993). Further maxima were detected by Saar *et al* (1995).

To understand the oscillatory behaviour of the correlation function of clusters of galaxies we have calculated the correlation function for randomly and

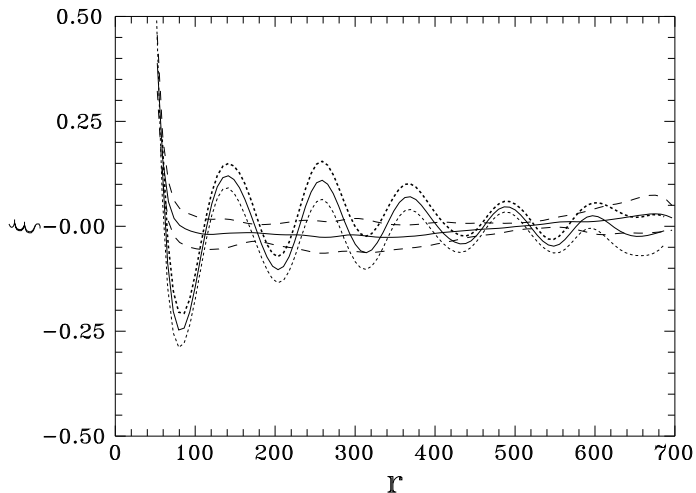


Figure 3: Correlation functions of clusters in geometrical models. The solid curve with dotted error corridor is the regular model with superclusters located along regularly spaced rods, the solid curve with dashed error corridor is the random supercluster model.

regularly located supercluster models. In the second model superclusters were located randomly along regularly spaced rods which form a rectangular lattice of step size $120 \pm 20 h^{-1}$ Mpc as samples of real rich superclusters. For each model we generated ten random realizations to calculate the error of the correlation functions from the scatter of these realizations.

The correlation functions for our geometric supercluster models are given in Figure 3. As expected, the correlation function for the regular model is oscillating as the correlation function of clusters in rich superclusters. The correlation function of the random supercluster model has a large positive correlation on small scales and zero mean correlation on large scales. Nearest neighbour test, and pencil-beam and void analysis indicate that clusters in poor superclusters with less than 8 members and isolated clusters form a population, preferentially located in void walls between rich superclusters but not filling the voids (Einasto *et al* 1994, 1997a). This analysis shows that on large scales the correlation function characterises the regularity of distribution of clusters of galaxies, differences between the random supercluster model and actual distribution of poor superclusters in void walls is irrelevant.

4 Power spectrum of clusters of galaxies

To calculate the power spectrum we have used the sample which contains clusters with measured redshifts only and lying in both galactic hemispheres out to

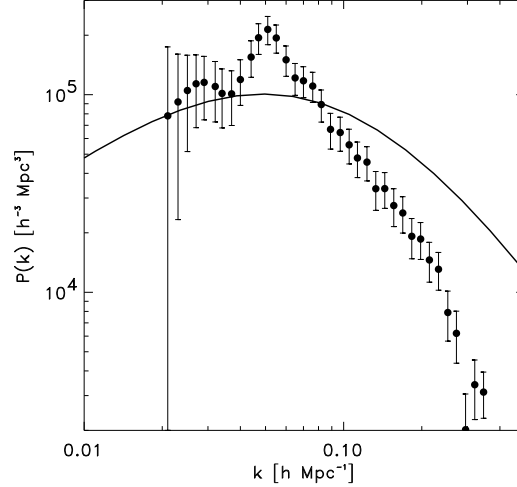


Figure 4: The power spectrum for 869 clusters with measured redshifts is plotted with solid circles. The spectrum is calculated from the cluster correlation function via the Fourier transform. Errors were determined from 2σ errors of the correlation function found from the scatter of different simulations. The solid line is the standard CDM ($h = 0.5$, $\Omega = 1$) power spectrum enhanced by a bias factor of $b = 3$ over the four year COBE normalisation.

the distance covered by our cluster and supercluster catalogues. The power spectrum was derived using two different methods, a direct one where we calculate the distribution of clusters in the wavenumber space, and an indirect method where we first calculate the correlation function of clusters of galaxies and then find the spectrum. In the latter case we make use of the fact that the power spectrum and the correlation function are related by the Fourier transform.

In both methods the main problem is the calculation of the selection function of clusters of galaxies which corrects for incompleteness both at low galactic latitude b and at large distances r from the observer. The selection function can be represented by linear functions of $\sin b$ and r (Einasto *et al* 1997a, 1997b). Both methods to derive the power spectrum yield similar results. However, parameters of the correlation function are less sensitive to small inaccuracies of the selection function, thus the indirect method yields more accurate results for the power spectrum.

To check the indirect method of calculation of the power spectrum we have used simulated cluster samples having similar selection effects as real ones. This check was performed for a wide variety of models with different initial spectra. Our results show that the true spectrum can be restored over the wavenumber interval from $k \approx 0.03 h \text{ Mpc}^{-1}$ towards shorter waves until $k \approx 0.3 h \text{ Mpc}^{-1}$.

The power spectrum for clusters of galaxies is shown in Figure 4. On very large scales the errors are large due to incomplete data. On moderate scales

we see one single well-defined peak at a wavenumber $k_0 = 0.052 \ h \text{ Mpc}^{-1}$. Errors are small near the peak, and the relative amplitude and position of the peak are determined quite accurately. The wavelength of the peak is $\lambda_0 = 2\pi/k_0 = 120 \pm 15 \ h^{-1} \text{ Mpc}$. Near the peak, there is an excess in the amplitude of the observed power spectrum over that of the CDM model (see below) by a factor of 1.4. Within observational errors our power spectrum on large scales is compatible with the Harrison-Zeldovich spectrum with constant power index $n = 1$, and on small scales with a spectrum of constant negative power index $n = -1.8$.

Our calculation show also that spectra found for the sample of all clusters and for the sample of clusters located in rich superclusters are very similar, only the amplitude of the spectrum of clusters in rich superclusters is higher. The sample of clusters in all supercluster richness classes is larger than the rich supercluster sample and random errors of the spectrum are smaller. The sample of all clusters (with and without measured redshifts) is still larger but in this case distance errors distort the correlation function a bit more and the spectrum is less certain. For this reason we have used the spectrum for all clusters with measured redshifts.

5 Correlation functions and spectra in N-body models

The power spectra and correlation functions are often used to compare the distribution of matter in the Universe with theoretical predictions.

We have calculated several models of structure formation using the standard PM code with 128^3 particles and 256^3 cells. Periodic initial conditions were used in the computational volume of side-length $L = 768 \ h^{-1} \text{ Mpc}$. Four initial spectra were used, corresponding to the standard CDM scenario with $\Omega_0 = 1$ and Hubble constant $h = 0.5$, a CDM model with cosmological constant ($\Omega_\Lambda = 0.7$, $\Omega_0 = 0.3$), a double power-law model with spectral index $n = 1$ on large scales, and index $n = -1.5$ on small scales, and a transition at scale $\lambda_t = 128 \ h^{-1} \text{ Mpc}$, and a similar double power-law model with an extra peak near the maximum of the spectrum. Clusters of galaxies were selected using a friend-of-friends algorithm for test particles representing the clustering of dark matter particles. The cluster power spectrum of matter for the standard CDM model is shown in Figure 4.

Currently popular structure formation theories are based on the dynamics of a Universe dominated by Cold Dark Matter. Spectra of CDM-type models are rising on long wavelengths λ (small values of the wavenumber $k = 2\pi/\lambda$), and falling on small wavelengths (large values of k). The transition between small and long wavelength regions in the CDM-spectra is smooth. The distribution of superclusters in CDM-models is irregular (Frisch *et al* 1995).

Spectra of double power-law models have a sudden transition between small and long wavelength regions. Correlation functions of clusters of galaxies of these models are oscillating. The amplitude of oscillations is very large in the model with an extra peak near the maximum of the spectrum. Rich superclusters in double power-law models form a moderately regular network, the regularity is strong in the model with extra peak in the spectrum.

The relative amplitude of the observed power spectrum above the standard CDM-type model is not very large. Thus we may ask the question: within the framework of the standard CDM-cosmogony, how frequently can we expect to find a distribution of clusters which has a power spectrum and correlation function similar to that observed? To answer this question we determined the correlation function and power spectrum for clusters in rich superclusters of CDM-type models. In the spectral range of interest the power spectrum of the standard CDM model is similar to the spectrum of a random supercluster model (Einasto *et al* 1997c). In both cases the correlation function of rich superclusters in double conical volumes has randomly located peaks and valleys. We have generated 1000 realizations of the random supercluster model, applied the selection effects as found in cluster distribution, and determined the parameters of the cluster correlation function and power spectrum. To quantify oscillating properties of the correlation function we measured the mean period and amplitude and their respective scatter. We also calculated the deviations for individual periods. This test shows that combination of parameters close to the observed values occurs in approximately 1 % of cases, but the simultaneous concurrence of all parameters with observations is a very rare event (of the order of one in million). Thus some change in the initial spectrum of matter is necessary in order to explain the observed correlation function and power spectrum for clusters of galaxies.

6 Conclusions

Our study of the distribution of clusters of galaxies has lead us to the following main conclusions.

- The distribution of high-density regions in the Universe (rich superclusters) is more regular than expected previously. Superclusters and voids form a cellular lattice or network with step size $120 \pm 20 h^{-1}$ Mpc. The location of cells is rather regular.
- The correlation function of clusters of galaxies has an oscillatory behaviour with regularly spaced secondary maxima and minima. The period of oscillations, $120 h^{-1}$ Mpc, is equal to the scale of the supercluster-void network. The power spectrum of the cluster correlation function has a sharp peak on the respective wavelength.

- Clusters of galaxies in CDM-type models of structure formation are located less regularly than real clusters.
- If the distribution of clusters of galaxies reflects the distribution of all matter then presently popular structure formation theories need revision.

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